DISCUSSION PAPER

Capacity Constraints, Price Discrimination, Inefficient Competition and Subcontracting

Matthias Hunold,
Johannes Muthers

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Abstract

We characterize mixed-strategy equilibria in a setting with capacity constrained suppliers which can charge location based prices to different customers. The equilibrium prices weakly increase in the transport distance between supplier and customer, whereas the margins decrease. Despite prices above costs and excess capacities, the competing suppliers exclusively serve their home markets in equilibrium. Competition yields volatile market shares and an inefficient allocation of more distant customers to firms. Even ex-post subcontracting may restore efficiency only partly. The suppliers sometimes do not cross-supply each other as this can intensify competition by relaxing the receiver's capacity constraint. We use our findings to discuss recent competition policy cases and provide hints for a more refined coordinated-effects analysis.

JEL classification: L11, L41, L61
Keywords: Bertrand-Edgeworth, capacity constraints, inefficient competition, spatial price discrimination, subcontracting, transport costs.

1 Introduction

The well-known literature based on Bertrand (1883)–Edgeworth (1925) studies price competition in case of capacity constraints – but does so mostly for homogeneous products. A recent example is Acemoglu et al. (2009). We contribute at a methodological level with a model where capacity constrained firms are differentiated by their costs of serving different customers and can charge customer specific prices. This leads to mixed price strategies with different prices for different customers, which is a new and arguably important element in this strand of literature. One novel aspect of our analysis is that the mixed strategies induce cost-inefficient supply relations such that transport costs are not minimized.

Various competition policy cases indeed feature products with significant transport costs for which location or customer based price discrimination is common. There are also merger control decisions in relation to such products that use standard Bertrand-Edgeworth models, which unfortunately do not take geographic differentiation and customer specific pricing into account. For instance, in the assessment of the merger M.7009 HOLCIM / CEMEX WEST the European Commission argued "*that the most likely focal point for coordination in the cement markets under investigation would be customer allocation whereby competitors refrain from approaching rivals' customers with low prices*". Moreover, it reasoned that "*given the low level of differentiation across firms and the existing overcapacities, it is difficult to explain the observed level of gross margins as being the result of competitive interaction between cement firms.*" As a supporting argument, the European Commission referred to a Bertrand-Edgeworth model with constant marginal costs and uniform pricing.[1]

Our model makes several predictions which can be related to the above reasoning: Even with overcapacities of 50%, we find that in a competitive equilibrium firms may always serve their nearest customers ("home market"), and that at prices above the costs of the closest competitor. Firms set high prices in the home markets of rival firms, although a unilateral undercutting there seems rational in view of their overcapacities. Such a pattern is difficult to reconcile with previous models of competition. To answer the question whether firms are indeed coordinating or competing, our model – which allows for geographic differentiation, location specific pricing and capacity constraints at the same time – could therefore improve the reliability of competition policy assessments.

As firms play mixed strategies in prices, we find that customers in between the firms are sometimes served by the firm with the lowest costs, and sometimes by the firm with higher costs. This result of inefficient competition arises in a symmetric setting with efficient rationing, where each firm has sufficient capacity to serve its closest customers and where firms can price discriminate according to location. There thus seem to be enough instruments to ensure that prices reflect costs and the intensity of competition for each customer. There is also complete information about the parameters of the game, which means that the allocative inefficiency arises purely due to strategic uncertainty: As the one competitor does not know which prices the other competitor will ultimately charge in equilibrium, the less efficient firm sometimes wins the customer. This natural insight that price competition can be chaotic

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[1]See section 8 for a more detailed discussion and references.

due to strategic uncertainty and thus costly absent any frictions is, to our knowledge, very rarely reflected in formal models.

The allocative inefficiency provides a rationale for cross-supplies between the competing suppliers. There is scope for such subcontracting when one supplier makes the most attractive offer to a customer, while another one has free capacity to serve that customer at lower costs. Cross-supplies can be observed in various industries. For instance, see Marion (2015) for a recent article on subcontracting in highway construction. We show that cross-supplies can lead to an efficient production in certain situations, but not in all. Firms refrain from subcontracting when this frees up capacity of a constrained firm that has set low prices – as the additional capacity can increase competition on (otherwise) residual demand segments of the market. We also show that subcontracting can increase customer prices when the cross-supplier has a weak bargaining position towards the receiver.

The remainder of the article is structured as follows. We discuss the related literature in the next section, introduce the model in section 3, sketch the reference cases of local monopolies and unrestricted capacities in section 4, analyze competition with capacity constraints in section 5, and introduce subcontracts in section 6. In section 7 we endogenize the capacities and demonstrate that excess capacity occurs in equilibrium when firms optimally choose their capacities and demand is uncertain. We conclude in section 8 with a further discussion of inefficient competition, subcontracting, and Bertrand-Edgeworth arguments in competition policy.

2 Related literature

This article contributes to several strands of the existing literature. One strand addresses spatial price-discrimination. In a seminal article Thisse and Vives (1988) investigate the choice of spatial price policies in case of transport costs, but absent capacity constraints. They find that firms do not individually commit to mill pricing (which means that each customer pays the same free-on-board price plus its individual transport costs), but prefer to charge each customer prices according to the intensity of competition for that customer.[2] This results in a pattern of high prices for customers in the firms' home markets and low prices for customers in between the firms. We find that also with capacity constraints the equilibrium prices do not correspond to the simple mill pricing pattern. However, the pricing patterns with capacity constraints more closely follow the firms' own marginal costs and not only the intensity of competition.

There are a few articles and working papers which introduce differentiation in the context of capacity constrained price competition, notably Canoy (1996), Sinitsyn (2007), Somogyi (2016) as well as Boccard and Wauthy (2016). Canoy investigates the case of increasing marginal costs in a framework with differentiated products. However, he does not allow for customer specific costs and customer specific prices. Somogyi considers Bertrand-Edgeworth competition in case of substantial horizontal product differentiation in a standard

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[2]This is essentially determined by the second most efficient firm's marginal cost for each customer.

Hotelling setting. Boccard and Wauthy focus on less strong product differentiation in a similar Hotelling setting as Somogyi. Somogyi uses a logit demand specification and shows that pure-strategy equilibria exist for small and large overcapacities, but only mixed-strategy equilibria for intermediate capacity levels. For some of these models equilibria with mixed-price strategies over a finite support exist (Boccard and Wauthy (2016); Sinitsyn (2007); Somogyi (2016)). This appears to be due to the combination of uniform prices and demand functions which, given the specified form of customer heterogeneity, have interior local optima as best responses.[3] Overall, these contributions appear to be mostly methodological and partly still preliminary.

The present article is also related to the literature on subcontracting relationships between competitors (also referred to as cross-supplies). With a subcontract, a firm can essentially use the production capabilities of a competitor. Efficiencies can, for instance, arise when a firm with decreasing returns to scale has won a large contract, so that subcontracting part of the production to an identical firm reduces costs (Kamien et al. (1989)). Similarly, if there are increasing returns to scale, pooling the production can reduce costs (Baake et al. (1999)). More indirectly, if firms with asymmetric costs compete, the resulting allocation of demand to each firm may not exactly minimize costs, such that again subcontracting increases efficiency (Spiegel (1993)).

The above literature on subcontracting has essentially pointed out two competitive effects, which depend on how the efficiency rents are shared between the two parties to the subcontract. If the receiver obtains the efficiency rent, its effective costs are lower as it uses the partly more efficient production technology of its competitor at costs. This tends to increase competition. Instead, if the cross-supplier obtains the efficiency rent, it foregoes a profit when competing as that reduces the probability to make rents on subcontracting. This tends to soften competition.[4] For instance, Spiegel (1993) points out that ex-ante agreements to cross-supply, which are concluded before firms compete, tend to dampen competition.

We contribute to this literature on subcontracting in several ways. For this, we focus on ex-post subcontracting, which takes place after firms have competed in prices. First, we point out that horizontal subcontracting may also occur when firms are symmetric and there are no generic reasons for subcontracting. In particular, if there was a symmetric equilibrium in pure price strategies, there would be no scope for subcontracting. The only reason for subcontracting is that price competition with capacity constraints can be so chaotic that the resulting allocation of customers to firms does not correctly reflect the differences in production costs, although customer specific pricing is feasible. We show that subcontracting can increase or decrease consumer surplus, depending on the distribution of the efficiency gains among the subcontracting competitors. Moreover, we show that there is a – to our

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[3]In a related vein, some work has considered heterogeneous consumers in models with different consumer segments where mixed-strategy price equilibria result, see Sinitsyn (2008, 2009), which is in line with the Varian (1980) model of sales.

[4]Marion (2015) finds that in California highway construction auctions the winning bid is uncorrelated with horizontal subcontracting and attributes this to an efficiency motive for cross-supplies. See also Huff (2012) for a similar work.

knowledge – new reason for why firms may not engage in welfare-improving subcontracting. When a firm that produces at its capacity limit asks an unconstrained firm for a cross-supply to a customer which that firm can supply more efficiently, the unconstrained firm may deny this. The reason is that such a supply would leave the demanding firm with additional capacity, which can intensify the competition for other customers.

3   Model

Set-up

There are two symmetric firms. Firm L is located at the left end of a line, and firm R at the right end of this line. Four customers are located on the line, with names 1, 2, 3 and 4 from left to right. The firms produce homogeneous goods. Each customer has unit demand and values the good at v. Firm L incurs transport costs of $1c$, $2c$, $3c$, and $4c$ to reach the customers from left to right. For firm R, there are transport costs of $4c$, $3c$, $2c$, $1c$ to reach the same customers. There are no other costs of supply. We assume that the value of the good is higher than the transport costs even for the most distant customers: $v > 4c$, so that each customer is contestable. See figure 1 for an illustration.
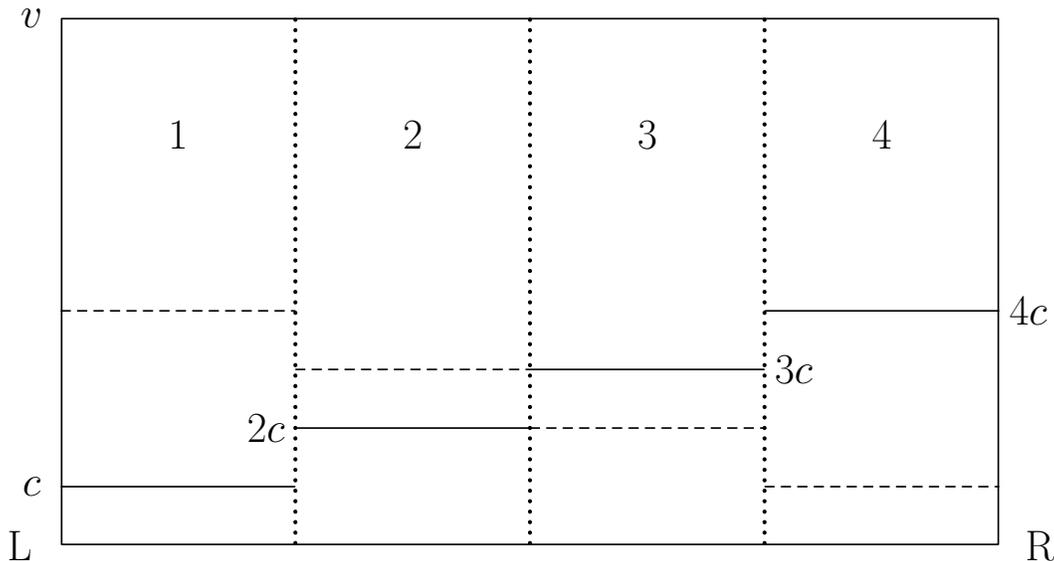
Figure 1: Setting: Customers 1 to 4 with unit demand and willingness to pay of v are located between firms L and R; the transport costs increase in the distance to each customer and range from c to $4c$.

Each firm $j \in \{L, R\}$ can price discriminate between customers by charging a separate price p_i^j to each customer $i \in \{1, 2, 3, 4\}$. A pure price strategy for firm j is a vector $(p_1^j, p_2^j, p_3^j, p_4^j) \in R^4$. We solve the game from the perspective of firm L and apply symmetry. If we suppress superscript j, p_i belongs to firm L.

The game has the following timing:

1. Firms L and R simultaneously set the eight prices p_i^L and p_i^R, $i \in \{1, 2, 3, 4\}$;

2. Customers are allocated to firms – according to prices and capacity constraints.

As a tie-breaking rule, we assume if both firms charge a customer the same prices, the customer buys from the firm with the lower transport costs. In our main case each firm has capacity to serve up to three of the four customers. As a consequence, a single firm cannot serve the whole market, whereas overall there is 50% overcapacity.

A small discrete number of customers is a simplification, but we conjecture that this is without significant loss of generality. For instance, consider a continuous distribution of customers on a unit line, as in the Hotelling model, but with firms bearing the transport costs. Still, the four customers would correspond to the market allocations that can arise if each firm charges a uniform price to all customers and has a capacity to cover $3/4$ of the market. In particular, for equal prices each firm would serve half the market (two customers in the discrete case), while if one firm has a lower uniform price it would serve $3/4$ of the market (3 customers), and the other firm the residual demand of $1/4$ (one customer).

Rationing

We employ efficient rationing, in particular, we use the following rationing rule:

1. If one firm charges lower prices to all customers than the other firm and does not have capacity to serve all customers, we assume that the customers are allocated to firms so that consumer surplus is maximized. In other words, the consumer with the best outside option is rationed.

2. If the first point does not yield a unique allocation, the profit of the firm which has the binding capacity constraint is maximized (this essentially means cost minimization).

While this is not the only rationing rule possible, we consider this rule appropriate because:

- The rationing rule corresponds to the usual efficient rationing (as, for instance, used by Kreps et al. (1983)) in that the customers with the highest willingness to pay are served first. A difference is, however, that the willingness to pay is endogenous in that it depends on the (higher) prices charged by the other firm. These may differ across customers, and so does the additional surplus for a customer from purchasing at the low-price firm.

- At least for the case of uniform prices ($p_1^j = p_2^j = p_3^j = p_4^j$), other rationing rules yield the same outcome. In particular, the firm with lower prices would also choose to serve the closest three customers, as this minimizes the transport costs.

- The rationing rule gears at achieving efficiencies, in particular for equilibria in which the firm's prices weakly increase in the costs of serving each customer. Our results of inefficiencies in the competitive equilibrium are thus particularly robust.

- The rationing rule is the natural outcome if the customers can coordinate their purchases: They will reject the offer that yields the lowest customer surplus. This occurs, for instance, if interim-contracts with side payments among the customers are allowed. It would also occur if there is only one customer with production plants at different locations.

- If a firm has to compensate a customer to which it made an offer that it cannot fulfill, this might also incentivize the firm to ration according to the customer's net utility from this contract.

In the next sections we solve the price game for Nash equilibria in differentiated prices. Whenever firms are symmetric, we focus on symmetric equilibria. We start by characterizing symmetric Nash equilibria for two reference cases: each firm can serve only half of the market, and each firm has capacity to supply the whole market. Afterward, we analyze the most interesting case when each firm has an intermediate level of 3 capacity units.

4 Reference cases

Pure strategy equilibrium when each supplier can serve half of the market

Suppose that each firm has capacity to serve only 2 out of the 4 customers. As a consequence, it is an equilibrium in pure strategies that each firm sets the price for each customer equal to the willingness to pay that equals v, and that each customer buys the good from the closest firm. This is efficient as all customers are served by the firm with the lowest transport costs. Each firm obtains the highest profit that is feasible with two units of capacity, which equals $2v - 3c$. Customer surplus is given by $4v - 4v = 0$.

Pure strategy equilibrium without capacity constraints

Suppose that each firm has capacity to serve all 4 customers. As a consequence, for each customer the two firms face Bertrand competition with asymmetric costs. It is thus an equilibrium in pure strategies that each firm sets the price for each customer equal to the highest marginal costs of the two firms for serving that customer, and that the customer buys the good from the firm with the lower marginal costs.[5] This is again efficient (for given capacities) in that all customers are served by the closest firm with the lowest transport costs. Each firm makes a margin of $4c - c = 3c$ from selling to the closest customer, and $3c - 2c = c$ from selling to the second closest customer. The equilibrium profit of a firm is thus $4c$. Consumer surplus is given by $4v - 2 \cdot 4c - 2 \cdot 3c = 4v - 14c > 0$.

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[5]We employ the tie breaking rule that the firm with the lower costs serves a customer if both firms charge the same price.

5 Competition with capacity constraints

Non-existence of a pure strategy equilibrium

Suppose each firm can only serve at most 3 customers and both firms set prices as if there were no capacity constraints, as discussed in the previous subsection. Is this an equilibrium? For each firm, the prices charged to its two most distant customers equal its costs of supplying each of these customers ($3c$ and $4c$). Hence, there is no incentive to undercut these prices. Similarly, there is no incentive to reduce the prices for the two closest customers as these customers are already buying from the firm.

In view of the other firm's capacity constraint, the now potentially profitable deviation is to set the highest possible price of v for each customer. All customers then prefer to buy from the other firm at the lower prices, which range between $3c$ and $4c$. However, as each firm only has capacity to serve 3 customers, one customer ends up buying from the deviating firm at a price of v. Given the rationing rules, this is the closest customer as the price of the other firm is largest for that customer. The profit of the deviating firm is thus $v - c$. This is larger than the pure strategy candidate profit of $4c$ if $v > 5c$.

The above condition for a profitable deviation is more restrictive by one c than the contestability assumption $v > 4c$. For the range $5c > v > 4c$ the equilibrium in pure strategies as in the case without capacity constraints also exists when each firm has 3 units of capacity. It is also the most reasonable candidate equilibrium here, as in any symmetric equilibrium both firms have one free unit of capacity, such that any price of the competitor above the own marginal costs can be profitably undercut.[6]

Mixed strategy equilibria

We now focus on the case that $v > 5c$ and solve the price game for symmetric mixed strategy Nash equilibria. Such an equilibrium is defined by a symmetric pair of joint distribution functions over the four prices of each firm. We proceed by first postulating properties and then derive results that hold for any equilibrium that has these properties. In the last step we verify that the initially postulated properties hold in equilibrium. With this approach we are able to show that an equilibrium with the following properties exists, however, we do not exclude that mixed strategy equilibria with other properties may also exist.

Properties of the equilibria. Both firms play mixed price strategies that are symmetric across firms with prices that are weakly increasing in distance: $p_1^L \leq p_2^L \leq p_3^L \leq p_4^L$ and $p_4^R \leq p_3^R \leq p_2^R \leq p_1^R$. Every individual realization of each player's price vector in the mixed strategy equilibrium has this price order. Moreover, each individual price is mixed over the same support $[\underline{p}, v]$ and there are no mass points in the marginal distributions of the prices.

Note that weakly increasing prices imply that the joint distribution does not have full

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[6]There are, however, potentially equilibria with even lower prices, in which firms set prices below for customers that are closer to the competitor. We exclude those equilibria as it is usual in the literature on asymmetric Bertrand competition.

support as certain price combinations are ruled out. We first provide some base results that hold for all equilibria with the above defined characteristics. We start with a property for the sales allocation, which is derived from the postulated property that firms play only weakly increasing price vectors.

Lemma 1. *If both firms play weakly increasing prices, there is zero probability that any firm serves the most distant customer, whereas the closest customer is served with probability 1.*

Proof. There are two cases to distinguish, either the capacity constraint of a firm is binding, or not binding:

1. All the prices of one firm are below the prices of the other firm: the firm with the lowest prices serves its closest three customers up to the capacity limit; the most distant customer is served by the firm with the high prices. This minimizes the prices that are charged and thus maximizes customer surplus, in line with the rationing rule.

2. Instead, if prices intersect (each firm has the lowest price for at least its closest customer), the customer closest to each firm is be won by that firm as there is no rationing.

$\square$

We now establish that a price vector with $p_1^L < p_2^L$ cannot be a best response to a distribution of weakly increasing price vectors played by the other firm.

Lemma 2. *In any symmetric equilibrium with weakly increasing prices, the prices for the two closest customers are identical: $p_1^L = p_2^L$, and by analogy $p_4^R = p_3^R$.*

Proof. There are two cases to distinguish:

First, suppose that $p_2^L \leq p_2^R$. Given a weakly increasing price order of firm R, firm L wins the first customer with a price of $p_1^L = p_2^L$ as $p_1^R \geq p_2^R \geq p_2^L$. Consequently, $p_1^L \leq p_2^L \leq p_1^R$. Hence, there is no incentive to charge a price $p_1^L < p_2^L$, as $p_1^L = p_2^L$ ensures a higher margin without losing demand.

Second, suppose that $p_2^L > p_2^R$. Given weakly increasing price orders, this means that R also has the lowest prices for customers 3 and 4. In this case firm L will serve customer 1 even if it has a higher price than R, as R – given the rationing rule – serves its three closest customers, such that customer 1 only has the option to buy from L or not at all. In this case setting $p_1^L < p_2^L$ is strictly worse than $p_1^L = p_2^L$.

In both cases the price relation $p_1^L < p_2^L$ – and by analogy $p_4^R < p_3^R$ – is strictly worse and thus dominated by equal prices for the two closest customers, which establishes the lemma. $\square$

Recall that when suppressing the superscript we mean prices of firm L. Weakly increasing prices imply that if the price for the closest customer is at the maximal level, $p_1 = v$, the other three prices of firm L must also be equal to v, such that $p_1 = p_2 = p_3 = p_4 = v$ is played with positive joint density. Similarly, the lower bound $\underline{p}$ is played with positive

density, which implies that $p_4 = \underline{p}$ is played in with positive density. Again, if $p_4 = \underline{p}$ is played, weakly increasing prices imply $p_1 = p_2 = p_3 = p_4 = \underline{p}$ is played with positive joint density. Thus uniform prices are played with positive density, at least at the boundaries of the price support.

Hence, for all customers the upper bound price v is played with positive joint density. In turn, we can exclude that there are mass points at v. By symmetry, only symmetric mass points have to be considered. However, each firm would want to deviate from a symmetric mass point by moving joint density to a price that is just below v, thereby increasing demand at virtually no costs. A firm can thus realize a profit of $\pi(v) = v - c$ with probability one by choosing a price of v for all customers. As all price vectors played in equilibrium must yield identical expected profits, the expected profit for each price vector that is played with positive density must equal $v - c$. This must also hold for the uniform price vector with the lowest price $\underline{p}$, which is played with positive joint density – as argued before. This yields the profit identity $\pi\left(\underline{p}\right) = \pi(v)$, which yields $\left(\underline{p} - c\right) + \left(\underline{p} - 2c\right) + \left(\underline{p} - 3c\right) = v - c$, and defines the lowest price

$$\underline{p} = \frac{1}{3}v + \frac{5}{3}c. \tag{1}$$

Lemma 3. *In any symmetric mixed strategy equilibrium with only weakly increasing prices and support $[\underline{p}, v]$ for all marginal price distributions, uniform prices are played with positive density at $\underline{p}$ and v.*

In the next subsection we construct and verify the existence of equilibria with only uniform prices, and then continue with equilibria that exhibit increasing prices.

Mixed strategy equilibria with uniform prices

In this subsection we construct equilibria in which both firms play only uniform prices. This is a special case of an equilibrium in weakly increasing prices. We first assume that firms are restricted to uniform price vectors and derive the equilibrium distribution functions. We then check for which parameter range it is an equilibrium that firms play only these uniform price distributions, even though they are allowed to play non-uniform price vectors.

If firms play uniform prices, all prices have the same marginal distribution function $F_1^j = F_2^j = F_3^j = F_4^j \equiv F$ for $j \in \{L, R\}$ by construction. Using the results of lemma 1, the expected profit of firm L can be expressed as

$$\pi^L = (p_1 - c) + \left[1 - F_2^R(p_2)\right](p_2 - 2c) + \left[1 - F_3^R(p_3)\right](p_3 - 3c).$$

For uniform prices $p_1 = p_2 = p_3 = p_4 \equiv p$ the profit becomes

$$\pi^L = (p - c) + [1 - F(p)](p - 2c) + [1 - F(p)](p - 3c). \tag{2}$$

Let us first characterize the distribution function F by using that firm L must be indifferent between all uniform prices that it plays with positive density.

Lemma 4. *If we restrict strategies to uniform prices, it is an equilibrium that firms mix uniform prices according to the distribution function*

$$F(p) = \frac{3p - 5c - v}{2p - 5c} \qquad (3)$$

on the support $[\underline{p}, v]$. The expected equilibrium profit equals $v - c$ and there is an expected cost inefficiency of c.

Proof. Using that the expected profit for any price vector that is played in equilibrium must be $v - c$, we can solve for $F(p)$ in the symmetric equilibrium in uniform prices by equating the profit from (2) with $v - c$:

$$(p - c) + [1 - F(p)](p - 2c) + [1 - F(p)](p - 3c) = v - c$$

$$\Leftrightarrow F(p) = \frac{3p - 5c - v}{2p - 5c}.$$

As almost surely the prices of the firms are not equal, there is an inefficiency of c as one of the two intermediate customers 2 and 3 is served by a firm with transport costs that are higher by c than those of the more efficient firm, which in these cases has set higher prices and still has unused capacity. □

So far we have characterized F such that the competitor is indifferent between all uniform price vectors within the support. Let us now check whether profitable deviations are possible with strictly increasing price vectors. We later on verify that there are no profitable deviations with price vectors that are not weakly increasing. We have already established that changing p_4 does not change profits as long as the price order is maintained (see lemma 1), and that $p_1 = p_2$ is optimal (see the proof to lemma 2). Hence, we need only to verify that there is no incentive to change p_3 individually or p_1 and p_2 together, which yields:

Lemma 5. *If $v \geq 7c$ and if price vectors are restricted to weakly increasing prices, there is a symmetric equilibrium in uniform prices with price distribution F. Instead, for a lower willingness to pay relative to the transport costs ($7c > v > 5c$), uniform prices cannot be an equilibrium.*

Proof. As established in lemma (2), a best response to weakly increasing prices with weakly increasing prices has the property $p_1 = p_2 \leq p_3 \leq p_4$. As the most distant customer is never served, we restrict our search for best responses to price vectors with p_4 equal to p_3 (as $p_4 = p_3$ is always a best response). This leaves only one critical price step in the best responses: the potential step between prices p_2 and p_3. We first verify that there is no incentive to deviate from uniform prices by increasing the price for customer 3 individually, while maintaining the order of weakly increasing prices. We afterwards verify that only weakly increasing prices are best responses to uniform prices. Note that changing the price of customer 3 (and 4) in a way that the order of weakly increasing prices is maintained does not affect the expected profits of that firm with customers 1 and 2.

There is an underlying incentive for a firm to charge higher prices to more distant customers as these are more costly to serve. To see this, note that the expected profit for firm L from serving one customer $i \in \{1, 2, 3, 4\}$ with the lowest price (i.e. without residual demand profits and in the absence of capacity constraints) is given by $[1 - F(p_i)](p_i - i \cdot c)$. Differentiating with respect to p_i yields

$$[1 - F(p_i)] - f(p_i) p_i + f(p_i) \cdot i \cdot c. \tag{4}$$

The marginal profit for firm L increases in the distance i. There is thus a natural incentive to set higher prices for more distant customers. Hence, if there is no incentive to increase p_3, then, for the same price distribution played by the other firm, there is also no incentive to increase p_2 (and p_1). We evaluate the marginal profit in (4) for customer 3 by substituting $i = 3$, and for f and F from (3). From this we derive the parameters at which the marginal profit is negative, such that a marginal price increase of only p_3 is not profitable:

$$\left[1 - \frac{3p - 5c - v}{2p - 5c}\right] - \frac{2v - 5c}{(2p - 5c)^2}(p - 3c) < 0$$

$$\implies (2p - 5c)(v - p) - (2v - 5c)(p - 3c) \quad < \quad 0. \tag{5}$$

For $p = \underline{p}$ the marginal profit condition reduces to $-\frac{1}{3}v + \frac{7}{3}c < 0$. This is equivalent to $v > 7c$. Moreover, we show that the second derivative of the profit for customer 3 is negative in the relevant range. The second derivative is given by

$$2(v - p) - (2p - 5c) - (2v - 5c) \quad = 2v - 2p - 2p + 5c - 2v + 5c = -4p + 10c.$$

This second derivative is already negative at the lower bound price of $\underline{p}$ for the lowest possible value for v of $5c$, above which there are mixed strategy equilibria. It is also negative for all larger prices up to v. The profit function is thus strictly concave in the relevant range. This implies that whenever the marginal profit (4) is negative at $\underline{p}$, it is negative for the prices above $\underline{p}$. □

Showing in addition that a firm cannot profitably deviate from uniform prices with prices that are not weakly increasing, given the other firm only plays uniform prices, establishes

Proposition 1. *If the willingness to pay is at least 75% above the transport costs for the most distant customer ($v > 7c$), there exists a symmetric equilibrium in which the firms play uniform prices with distribution F in the support $[\underline{p}, v]$. The expected profit of a firm is $v - c$ and there is an expected cost inefficiency of c.*

Proof. Consider that firm R plays uniform prices. Suppose that firm L chooses prices that are not weakly increasing. In that case there is a pair p_j, p_k of prices of firm L with $p_k < p_j$ and $j < k$. In all such cases it is at least as profitable to switch p_k and p_j such that $p_k > p_j$. To see this, consider the three possible outcomes: the uniform price of R, p^R, is above, below, or in

between the price pair. Firstly, if $p^R > p_j > p_k$, switching p_j and p_k weakly increases profits. In particular, it is profit neutral if the capacity constraint is not binding, and strictly profit increasing if customer j is rationed, as this reduces the costs for the customers served without affecting the average price level of the customers that are served. Secondly, if $p_R < p_k < p_j$, switching p_k and p_j can affect which customer is served by firm L as its residual demand (if p_k is the lowest price) and is thus weakly profitable because serving customer j has lower costs. Thirdly, if $p_j > p_R > p_k$, the capacity constraint is not binding for either firm. In this case switching the prices to a weakly increasing price order is always profitable as it changes the customer that is served by firm L from k to j, with less costs and without changing the prices that are realized. □

It may not seem intuitive that firms charge uniform prices in equilibrium when costs differ and price differentiation is possible. However, note that a firm which faces a competitor that charges uniform prices can ensure to win the closest customer (its "home market") by not charging higher prices in its home market than to other customers. This allows to increase prices for the close customers up to the level of the distant customers. This incentive dominates the incentive to price through the cost differences when costs are sufficiently low, such that an equilibrium with only uniform prices exists. For sufficiently high costs, equilibria with only uniform prices cannot be sustained.

Mixed strategy equilibria with strictly increasing prices

So far we have established that for $v \leq 5c$ pure strategy equilibria exist, whereas for $v \geq 7c$ mixed strategy equilibria with uniform prices exist. Let us now investigate the parameter range $v \in (5c, 7c)$, where profitable deviations from uniform prices exist. Lemma 5 states that for this range each firm best responds to the uniform price distribution with strictly increasing prices in the sense of $p_3 > p_2$ in an interval starting at $\underline{p}$. We thus search for a price distribution which allows for strictly increasing prices.

The (expected) profit of firm L can again be written as

$$\pi_L = \left(p_1^L - c\right) + \left[1 - F_2^R\left(p_2^L\right)\right]\left(p_2^L - 2c\right) + \left[1 - F_3^R(p_3^L)\right]\left(p_3^L - 3c\right).$$

We have established that weakly increasing price vectors are of the form $p_1 = p_2 \leq p_3 \leq p_4$ (lemma 2). With weakly increasing prices, a firm never serves the most distant customer such that it is a best response to set $p_3 = p_4$. For marginal price deviations which maintain the weakly increasing price order, a firm is thus not capacity constrained with respect to the three closest customers. Moreover, a firm always serves its closest customer. Denoting $p_{12} = p_1 = p_2$, $p_{34} = p_3 = p_4$, $F_{12} = F_2^L = F_3^R$, and $F_{34} = F_3^L = F_2^R$, the profit of firm L becomes

$$p_{12} - c + (1 - F_{34}(p_{12}))(p_{12} - 2c) + (1 - F_{12}(p_{34}))(p_{34} - 3c). \tag{6}$$

Note that at the top of the price support, it must still be the case that the price of v is played with positive density for all customers (recall that we search for equilibria with full

support for the marginal distributions in $[\underline{p}, v]$). Weakly increasing prices imply that at the top of the price support, it must be that firm L sets $p_1 = p_2 = v$ only if $p_3 = p_4 = v$. In other words, firms must play uniform prices at the upper bound of the price support.

The largest possible interval in which uniform prices are a best response to uniform prices played with the distribution function F, as defined in (3), is given by $[p^I, v]$. The lower bound p^I is obtained by setting the marginal profit in (5) equal to zero and solving for p, which yields $p^I = \frac{1}{2}\left(5c + \sqrt{2cv - 5c^2}\right)$. Recall that uniform prices are constructed such that there is a marginal incentive to decrease p_3, which yields a corner solution at uniform prices, as decreasing p_3 below p_2 would overturn the order of weakly increasing prices. A further decrease yields a discrete profit decrease as now the customer that the firm serves when the other firm is rationed is a customer with discretely higher costs.

Consequently, at least for the complementary lower part of the price support, that is $\left[\underline{p}, p^I\right]$, uniform prices cannot be mutually best responses. As we demonstrate below, there are piecewisely defined marginal distribution functions in equilibrium. The marginal distributions differ between the two closest and the two most distant customers for prices from $\underline{p}$ up to a certain threshold, whereas firms play only uniform prices above that threshold up to prices of v. For any price range in which firms do not play uniform prices, each firm must be indifferent over all prices in that range when changing p_{12} or p_{34}. The marginal distribution functions F_{23} and F_{34} in the mixed strategy equilibrium are thus defined by an indifference condition. A marginal change in the prices $p_1 = p_2 = p_{12}$ should not change the profit:

$$1 + 1 - \underline{F}_{34}(p_{12}) - \underline{f}_{34}(p_{12})(p_{12} - 2c) = 0.$$

This condition defines the marginal distribution in the range of prices that are not only uniformly played. We denote this part of the distribution function by $\underline{F}_{34}$. The solution to the above differential equation is

$$\underline{F}_{34}(p) = \frac{2p - k_3}{p - 2c}.$$

At the lower bound price $\underline{p}$, it must be that the distribution function has a value of 0. This implies $k_3 = 2\underline{p} = \frac{2}{3}v + \frac{10}{3}c$ and thus

$$\underline{F}_{34}(p) = \frac{2p - \frac{2}{3}v - \frac{10}{3}c}{p - 2c}.$$

Non-uniform prices cannot be played globally as $\underline{F}_{34}(v) > 1$ for $v \in [5c, 7c]$.

To obtain $\underline{F}_{12}$, we differentiate the profit in (6) with respect to p_3 to obtain the marginal indifference condition

$$1 - \underline{F}_{12}(p_3) - \underline{f}_{12}(p_3)(p_3 - 3c) = 0.$$

The solution to this differential equation is

$$\underline{F}_{12}(p) = \frac{p - k_2}{p - 3c}.$$

Also, this distribution function must be 0 at $p = \underline{p}$, which implies $k_2 = \frac{1}{3}v + \frac{5}{3}c$ and yields

$$\underline{F}_{12}(p) = \frac{3p - v - 5c}{3p - 9c}.$$

Note that $\underline{F}_{12}(v) < 1$ for $v \in [5c, 7c]$, such that $\underline{F}_{12}$ can only describe part of the price distribution. However, the functions $\underline{F}_{12}(4c) = F_{12}(4c) = F(4c)$, where $4c$ is above the level p^I, above which uniform prices are feasible for the parameter range $7c > v > 5c$. This means that we can define marginal distribution functions F_{12} and F_{34}, where increasing prices are played from $\underline{p}$ to $4c$ and uniform prices from $4c$ to v, such that no firm wants to marginally deviate. In the symmetric candidate equilibrium, firms play prices that are consistent with $\underline{F}_{12}$ or $\underline{F}_{34}$ on the lower part of the support, and with F in the upper part. This yields well defined marginal distribution functions F_{12} and F_{34}.

In the symmetric equilibrium, we thus have marginal distributions for the prices p_1^L, p_2^L, p_3^R, and p_4^R of the two closest customers of

$$F_{12}\left(p_i^j\right) \equiv \begin{cases} \frac{3p_i^j - 5c - v}{2p_i^j - 5c} & \text{if } 4c < p_i^j \le v, \\ \frac{p_i^j - \frac{1}{3}v - \frac{5}{3}c}{p_i^j - 3c} & \text{if } \underline{p} \le p_i^j < 4c, \end{cases} \tag{7}$$

and for the prices p_3^L, p_4^L, p_1^R, and p_2^R of the two most distant customers of

$$F_{34}\left(p_i^j\right) \equiv \begin{cases} \frac{3p_i^j - 5c - v}{2p_i^j - 5c} & \text{if } 4c < p_i^j \le v, \\ \frac{2p_i^j - \frac{2}{3}v - \frac{10}{3}c}{p_i^j - 2c} & \text{if } \underline{p} \le p_i^j < 4c. \end{cases} \tag{8}$$

For $v = 7c$, the lower bound $\underline{p}$ equals $4c$ and the functions F_{12} and F_{34} coincide and equal F. This is consistent with our previous finding that uniform prices are played in equilibrium with the marginal distribution F on the whole support $[\underline{p}, v]$ if $v > 7c$ (proposition 1).

Showing that it is not profitable for a firm to set prices that are not weakly increasing in response to weakly increasing prices establishes

Proposition 2. *If $5c < v < 7c$, there exists a symmetric mixed strategy equilibrium with weakly increasing prices that satisfy $p_1 = p_2 \le p_3 = p_4$. Each firm mixes the prices for its closest two customers according to the price distribution F_{12} and for the two most distant customers according to F_{34}, as defined in (7) and (8). All marginal price distributions are atomless with support $[\underline{p}, v]$. The expected firm profit is $v - c$ and firms play strictly increasing prices with positive probability.*

Proof. Given the above arguments, it is left to verify that there is no profitable drastic deviation that overturns weakly increasing prices. Consider firm L for the argument. Given firm R plays weakly increasing prices with $p_1^R = p_2^R$ and $p_3^R = p_4^R$ according to the equilibrium

distributions F_{12} and F_{34}, we show next that every best response to that strategy has weakly increasing prices.

Suppose to the contrary that L violates weakly increasing prices by playing $p_j > p_k$ with $j < k$. Let us investigate the case $p_4 < p_3$. By the same logic as for uniform prices (proof to proposition 1), it is profitable to switch the two prices such that the high cost customer faces the higher price. As R sets identical prices for its two closest customers, only the same three outcomes of the uniform pricing case can occur. Moreover, the same logic holds for p_1 and p_2. Consequently, the price order is such that $p_1 \leq p_2$ and $p_3 \leq p_4$.

It is left to establish that $p_2 \leq p_3$. Let us first show that $p_2 \leq p_4$. Note that customers 2 and 4 cannot be the residual customer for L as customers 1 or 3 would be selected by the rationing rule, given that $p_1 \leq p_2$ and $p_3 \leq p_4$.

Suppose $p_4 < p_2$ and that the capacity constraint is binding for L (this occurs when L has lower prices than R for all customers). In this case either customer 4 or 2 is rationed (given $p_3 \leq p_4$ and $p_1 \leq p_2$). If customer 2 is rationed (which implies that L serves customer 4, but not 2), then it is profitable to increase p_4 to p_2 as this ensures that a higher price p_2 is realized at the lower costs for serving customer 2. If instead customer 4 is rationed, increasing p_4 has no effect on profits, whereas a lower p_4 reduces profits strictly, if it results in customer 2 being rationed (recall that $p_4 < p_2$ and costs are lower for customer 2). In summary, in this case there is a strict incentive to increase p_4 as long as it is not certain that customer 4 is rationed; once this is certain there is still a weak incentive.

Suppose $p_4 < p_2$ and that the capacity constraint of L is not binding. In that case increasing p_4 increases the expected profits of L. To see this, consider the marginal profit of L when changing p_4: As L faces equilibrium strategies of R which all have the property $p_4^R = p_3^R$, and $F_3^R = F_4^R$ and are designed such that L is indifferent over p_3 (the marginal profit (4) for $i = 3$ is zero), L has a strict incentive to increase p_4 for which it has larger costs (the marginal profit (4) for $i = 4$ is positive). Thus, there is an incentive to increase p_4 if it is below p_2 up to the point where it is certain that customer 4 is never served.

However, if it is certain that customer 4 is the one that is rationed in all situations where L is capacity constrained, the capacity constraint never binds for the first three customers. In that case p_3 is chosen according to the marginal condition. This condition ensures that L is indifferent over p_3 (the marginal profit (4) for $i = 3$ is zero), such that it is always a best response to increase p_3 up to p_2. This establishes that in response to the price distributions of the candidate equilibrium, it is not more profitable for L to set prices that are not weakly increasing. □

Note that proposition 2 does not explicitly refer to the joint price distribution of all 4 prices of a firm, but defines the price order for each draw and the marginal distributions. This is sufficient to characterize the equilibrium. This means that firms can use any joint distribution which fulfills the conditions characterized in proposition 2. As an illustration, we provide a consistent pricing rule in annex I.

Let us now discuss various properties of the equilibrium. The equilibrium profits are $v - c$, independent of a restriction to uniform prices. Intuitively, the same profit would result if we

restricted pricing to mill pricing (each customer pays the same free-on-board price plus its individual transport costs), as the residual demand profit would still equal $v - c$. By contrast, Thisse and Vives (1988) point out that the equilibrium profits without capacity constraints are typically higher if mill pricing is required. They argue that competition is relaxed as it is more difficult for the firms to individually poach each others' customers without sacrificing margin on "own" customers. See also Armstrong and Vickers (2001) for a discussion.

Without capacity constraints, there is asymmetric Bertrand competition for each customer, which is independent from the competition for other customers. This yields a spatial price pattern which reflects the intensity of competition between the firms. The intensity is highest where the difference in the firms' location specific transport costs is lowest. This is the case for the customers in the center (customers 2 and 3), which get the lowest prices.

By contrast, with capacity constraints, competition for different customers is linked through the capacity constraints – even when price discrimination is feasible. In case of a binding capacity constraint of one firm, the prices for the different customers charged by both firms determine which customer is rationed and allocated to the high-price firm. Each firm generally has an incentive to ensure that it serves its closest customer because this minimizes its transport costs. There is now an additional incentive to set the price for the closest customer not above the prices of other customers, as this ensures that the firm serves the closest customer, when the other firm becomes capacity constrained and rationing takes place. As the rationing rule accounts for customer surplus, charging a high price for the closest customer would bear the risk that a more distant customer with higher transport costs is allocated to the non-constrained firm in case of rationing. As a result, the spatial price pattern of a firm is more reflective of its transport costs than without capacity constraints – and thus prices tend to increase in distance.

Another effect of the capacity constraints is that the prices of all customers increase. This is because of the profit each firm obtains when serving a residual customer which cannot be served by a capacity constrained competitor, even if the competitor offers the lowest price to all customers. This extra profit affects the pricing for each customer – like an increase in marginal costs for all customers. This effect tends to reduce the differences between the prices charged to different customers, as – in relative terms – the differences in (opportunity) costs are reduced in case of capacity constraints.

Although firms are in their best responses indifferent between uniform and strictly increasing prices, in case of $5c < v < 7c$ the firms set on average lower prices for the two closer customers and higher prices for the two more distant customers. Stated differently, the prices can be ordered in the sense of first order stochastic dominance: $F_{12} \geq F_{34}$, with a strict inequality in the range $(\underline{p}, 4c)$, such that the price distributions of each of the closest two customers are first order stochastically dominated by the price distributions of the two most distant customers.

Cost inefficiency in equilibrium. Strategic uncertainty over prices causes a cost inefficiency of c when firm R sets a lower price for customer 2 than firm L because R then serves the customer with its higher transport costs (and the other way around for customer 3). The

probability that firm R sets a lower price for customer 2 is given by

$$Pr(p_2^R < p_2^L) = \int_{\underline{p}}^{v} \int_{\underline{p}}^{p_2^L} f_{34}(p_2^R) f_{12}(p_2^L) dp_2^R dp_2^L = \int_{\underline{p}}^{v} F_{34}(p_2^L) f_{12}(p_2^L) dp_2^L. \qquad (9)$$

Note that p_2^R and p_2^L are drawn independently as one is set by firm R, and the other by firm L. This implies that the joint density is the product of densities as shown in the intermediate step of (9). The probability that firm R sets a lower price is 50% in case of uniform prices. This means that in the interval $[4c, v]$ – where firms set uniform prices – the cost inefficiency occurs almost surely at either customer 2 or 3, where one supplier has the lower price for both customers, but only the lower cost for the closer customer. For the interval $[\underline{p}, 4c]$, firms set on average lower prices to their closer customers, in line with the lower transport costs. As a consequence, the probability of an inefficient supply is lower there. This means that the inefficiency is lower when strictly increasing prices are played, rather than only uniform prices. Price discrimination thus increases efficiency, but cannot fully achieve it as the cost inefficiency still occurs with positive probability.

6   Subcontracting

In the mixed strategy equilibria of section 5, the transport costs are inefficiently high. With positive probability a firm that has free capacity and is closest to a specific customer has not won a supply contract with that customer. There is thus scope for the firm that has won the customer to subcontract the delivery to the lower-cost firm. The firm that has initially won the contract would still charge the customer the agreed price and compensate the lower-cost firm for actually supplying the product. The resulting efficiency rent can be shared among the two firms. As a consequence, it seems that firms should always conclude such a subcontract.

In the established literature, if subcontracting is conducted after prices or quantities are set, it takes place whenever a cost saving is feasible (see section 2). We add to this literature the insight that even when cost-reducing cross-supplies are feasible after the firms have set their prices, they may nevertheless prefer to not engage in cross-supplies. The reason is that a cross-supply relaxes the capacity constraints of the firm that has set aggressive prices and thereby leads to lower equilibrium price realizations – even after prices have been set. The firms can thus be better off to not engage in efficiency-enhancing ex-post cross-supplies. We formally characterize this insight in the following extended game:

(i)   firms set customer prices – as before,

(ii)   firms see each others prices and can agree to cross-supply customers,

(iii)   customers are allocated to the firms – according to prices and capacity constraints.

In stage (ii) a firm anticipates that it will serve the customers for which it has the lowest price and free capacity. There is scope for a cross-supply if a firm has set the lowest price

for a more distant customer that could be served at lower costs by the other firm. If a cross-supply of a customer is agreed, the cross-supplier serves the customer from its location as a subcontractor, incurs the associated transport costs and receives a transfer from the low-price firm. The customer still pays the original price to the low-price firm.

Subcontracting yields an efficiency rent for the firms if a firm has won a customer for which it has the higher transport costs and the other firm has free production capacity. However, for the cross-supplier there is the potential disadvantage that the competitor receiving a cross-supply has one more unit of free capacity, which can be used to supply another customer. As an example, consider that firm R sets a uniform price of v for all customers, and firm L a strictly lower uniform price $p_L < v$. Without subcontracting, L will simply supply customers 1, 2 and 3 up to its capacity limit, and firm R serves the residual customer 4. Consider that firm R agrees to supply customer 3 by means of a subcontract with L. Now firm L has one more unit of free capacity. As a consequence, customer 4 will be allocated to firm L in stage (iii) as L charges a strictly lower price and – due to the subcontract for customer 3 – still has an unused unit of capacity. This implies that firm R would forego its residual demand profit of $v - c$. Indeed, the firms could agree to also subcontract for customer 4 to save the cost difference $4c - c$ as it is inefficient that firm L supplies the most distant customer 4. This cost saving is, however, only a side-effect of the cross-supply for customer 3, as otherwise firm L would be capacity constrained and firm R would serve customer 4 anyway – at its low transport costs of c. The only effective cost saving is thus that of $3c - 2c = c$ for customer 3. This needs to be traded off against the lost revenues when firm L serves customer 4 at a price of $p_L < v$ instead of firm R serving that customer at the higher price $p_R = v$. Taken together, a cross-supply can only yield a Pareto-improvement for the firms when the cost saving on customer 3 is higher than the lost revenue on customer 4: $c > p_R - p_L$.

Subcontracting reduces total costs by the cost difference between the two firms for each subcontracted customer. Depending on the expected payments between the firms, subcontracting changes the perceived cost when competing in stage (1). We follow Kamien et al. (1989) in assuming that the firms make take–it–or–leave–it offers. The firm that makes the offer has all the bargaining power as it can choose the terms of the contract such that the other firm does not get any additional rent. It can thus extract all the additional profits that subcontracting yields. We proceed by analyzing the two cases of either

(a)   the firm that has set the lower price and demands a cross-supply, or

(b)   the potential cross-supplier, i.e. the firm has set the higher price, but has lower costs for that customer,

making the offer and characterize the resulting equilibria.

For the analysis of subcontracting we abstract from price differentiation and only consider uniform price vectors. We have already established that without subcontracting uniform prices endogenously emerge for a large parameter range (proposition 1). With subcontracting, however, depending on timing and the bargaining power, different non-uniform price patterns might emerge and there appear to be multiple equilibria in each case. Abstracting from price

differentiation simplifies the analysis and comparison by yielding a unique equilibrium in each case. We conjecture that the results obtained for uniform prices qualitatively hold also for non-uniform prices.

(a) The firm demanding the cross-supply gets the additional rents

Consider that the firm which has set the lower uniform price offers the subcontract. It can make an offer that extracts the additional rents which arise from subcontracting. As a consequence, the perceived costs are lower because the two most distant customers can be efficiently supplied by the other firm with a subcontract. This is particularly relevant for the third closest customer, as without subcontracting the most distant customer is not served by the low-price firm due to its capacity constraints. Note that with uniform prices, if a firm supplies one unit as a cross-supply to the low-price competitor, it will not sell any unit itself – not even its home market. The reason is that the cross-supply frees capacity of the competitor, which enables it to serve the remaining customer for which it has also set the lowest uniform price. Hence, whenever a cross-supply is agreed, it contains two units, such that the firm with the lower price does not supply the two most distant customers itself.

With uniform prices played by the other firm according to the cdf $F(p)$, the expected profit of a firm is given by

$$\pi(p) = p - c + (1 - F(p))(p - 2c) + (1 - F(p))(p - 3c)$$
$$+ (F(p + c) - F(p)) \left(c - \int_p^{p+c} x \frac{f(x)}{(F(p + c) - F(p))} \, dx + p \right).$$

The last term is new here compared to the case without cross-supplies (see equation (2)). It states that with a probability of $(F(p + c) - F(p))$ the other firm sets a price in between p and $p + c$. In this case the cost saving of c on the third closest customer is larger than the foregone revenue of the cross-supplier on the most distant customer. The expected lost revenue for this case is the difference of the average price in the range p to $p + c$ and p.

Proposition 3. *If subcontracting takes place before rationing, and if the firm demanding the cross-supply gets the additional profits, and if only uniform prices can be played, there is a symmetric mixed strategy equilibrium with an atomless price distribution that has an upper bound of v. The expected profit in this symmetric equilibrium is $v - c$, customers benefit from subcontracting, total surplus increases, but the firms choose not to realize all feasible efficiency-enhancing cross-supplies.*

Proof. The existence of a symmetric mixed strategy equilibrium of the pricing game follows from theorem 6 in Dasgupta and Maskin (1986). The conditions are met as the action space is a compact and a non-empty subset of the real numbers, the sum of the firms' pay-offs is continuous, individual pay-offs are bounded, and weakly lower semi-continuous, with a strict inequality at the point of symmetry. Theorem 6 in Dasgupta and Maskin (1986) implies that

all possible points of discontinuity in the pay-off functions are atomless. As every price can be a point of discontinuity, the whole price distribution must be atomless.

Note that the sum of the firms' pay-offs is $3\min(p_L, p_R) + \max(p_L, p_R) - 7c$ without subcontracting and $4\min(p_L, p_R) - 6c$ in case of subcontracting. These profits are equal if the firms are indifferent between subcontracting and no subcontracting. Individual profits are weakly lower semi-continuous as they do not jump downwards within the price support, except where the prices of both firms are equal.

Price v is played with positive density. Suppose to the contrary that there was an upper boundary $\bar{p} < v$ in the symmetric mixed strategy equilibrium while there are no mass points in the distribution functions. The profit of such a price is $\bar{p} - c < v - c$, such that it is profitable to move density from $\bar{p}$ to v. The profit at a price of v is the residual demand profit $v - c$, as the receiver of a subcontract obtains the associated rent. As the profit must be the same on the whole support, the equilibrium profit is $v - c$.

Not all efficient cross–supplies take place as the maximal price difference, i.e., the range of prices in the support of the price distributions, is larger than c. To see this, note that the lowest possible price without subcontracting is $\underline{p}$ as defined in (1). The lowest price with subcontracting is even lower as the profit at the upper bound is still $v - c$, and the profit at the lower bound is higher, due to the additional profits from subcontracting. The price difference $v - \underline{p}$ is thus a lower bound for the range with subcontracting. The price range without subcontracting is already larger than c given the assumption $v \geq 4c$ because $v - \underline{p} = \frac{1}{3}(2v - 5c) \geq \frac{1}{3}(8c - 5c) = c$. Thus price differences larger than c occur with positive probability in case of subcontracting. Although it would increase efficiency, subcontracting does not take place in these cases because the firms loose more in revenues than they gain in cost reductions.

Given the atomless price distribution, cross-supplies take place with positive probability. Total surplus is higher as cross-supplies reduces the transport costs, whereas the expected profit per firm remains at $v - c$, equal to the profit without the efficiency increasing cross-supplies. As firms only engage in cross-supplies when their joint surplus increases, they must on average set lower prices for their expected profit to remain constant in spite of lower costs. This implies that customers benefit from cross-supplies through on average lower prices – at least when the firm demanding the cross-supply gets the associated rent. $\qquad\square$

(b) The cross-supplier gets the additional rents

Consider that the firm with the higher uniform price offers the subcontract. It can make an offer which extracts all the additional rents that arise from subcontracting. As a consequence, setting a high price becomes more attractive because in addition to obtaining the residual demand profit, an additional income from subcontracting arises with a positive probability. The expected equilibrium profit is thus larger when the cross-supplier determines the terms of the contract.

With uniform prices played by the other firm according to the cdf $F(p)$, the expected

profit of a firm is now given by

$$\pi(p) = p - c + (1 - F(p))(p - 2c) + (1 - F(p))(p - 3c)$$
$$+ (F(p) - F(p - c))\left(c + \int_{p-c}^{p} x \frac{f(x)}{(F(p) - F(p - c))}\, dx - p\right).$$

The last term is different compared to the case when the firm that has set the lower price gets the rents from subcontracting. It states that with a probability of $(F(p) - F(p - c))$ the other firm sets a price in between $p - c$ and p. In this case the cost saving of c on the second closest customer is larger than the foregone revenue on the closest customer (which is otherwise served by the firm that has set the higher price as residual demand). The expected lost revenue for this case is the difference of p and the average price in the range $p - c$ to p.

The expected profit of a firm choosing a price of v is

$$\pi(v) = v - c + (1 - F(v - c))\left(c + \int_{v-c}^{v} x \frac{f(x)}{(F(v) - F(v - c))}\, dx - v\right),$$

which defines the expected equilibrium profit. Note that the second term is the efficiency gain minus the positive externality on the customers, due to the price reduction for the closest customer. The size of that term increases in the density of prices close to v.

Proposition 4. *If subcontracting takes place before rationing, and if the cross-supplier gets the additional rents, and if only uniform prices can be played, there is a symmetric mixed strategy equilibrium with an atomless price distribution that includes v in the support of the price distribution. The expected profit of a firm in this symmetric equilibrium is between v and $v - c$, total surplus is higher than without subcontracting, but consumer surplus is lower. If $v > {}^9\!/_2 c$, not all efficiency-increasing subcontracts are realized.*

Proof. The first part of the proof with respect to the existence of a symmetric mixed strategy equilibrium and the characteristics of the profit function is virtually identical to that of proposition 3.

The highest price that is played with positive density is v. The expected profit at this price is bounded from above by $(v-c)+c = v$, where the first part is the residual demand profit and the second part the rent from subcontracting in the limit when the price of the competitor approaches v. The lower bound profit can be found by excluding the subcontracting profit, which yields additional profits for the firm with higher prices. The lower bound profit is thus $v - c$. The lowest price p' that is played must yield the same profit as the largest price v. To compute a lower bound for the price range, we obtain an upper bound of p' by equalizing the associated profit at this price with the upper bound profit at a price of v, that is $v = 3p' - 6c$ $\Leftrightarrow p' = v/3 + 2c$. Hence, the difference between the highest and the lowest price that are played with positive density is at least $v - p' = \frac{2}{3}v - 2c$, which is at least c if $v \geq 4.5c$. For this parameter range clearly not all efficient cross-supplies take place as the range of the support is larger than c.

Subcontracting increases total surplus at most by c, which is the efficiency gain if a

subcontract is realized each time. The total profits of both firms increase from $2(v - c)$ without subcontracting to at least the lower bound of $2v$. The increase in profits is thus at least $2c$. Consumer surplus is the difference between total surplus and firm profits and thus decreases by at least c. □

Irrespective of whether the cross-supplier or the receiver of the cross-supply gets the additional rents of the cross-supply, production becomes more efficient. Whether customers benefit through lower prices depends on the bargaining power among the subcontracting parties. If the firm that offers lower price is rewarded by being able to extract the cross-supplying rents, customer prices are lower than without subcontracting. If the firm with the higher price can extract the additional rents, prices are higher.

We have analyzed the case that firms subcontract after prices are set, but before rationing takes place. This is plausible as the cross-supplier change the available capacities of the firms. An interesting result is that firms do not in all cases of inefficient allocations agree on cross-supplies. The reason is that a firm that supplies a capacity constrained competitor which has set low prices has to fear that this competitor can serve additional customers once it has received the cross-supply.

If the allocation of customers to firms is instead finalized before firms can subcontract, all cost reducing cross–supplies will take place. This would correspond to a more static market, where capacities that are freed in the process of subcontracting cannot be brought back to the market.[7]

7 Endogenous capacities

In this section we investigate which capacity levels firms optimally choose before competing in prices. To focus on the strategic capacity trade-off, let us assume that capacity is costless, and that a firm chooses the lower level if two different levels yield the same profit.

Consider first that the level of demand is certain and common knowledge when firms choose capacities. As assumed so far, there are four customers, each with unit demand. Each firm can choose an integer capacity of $1, 2, 3$ or 4 units. It is never profitable for a firm to choose a capacity level that exceeds the total demand of 4 units as it can obtain the same profit with a capacity level of 4 – the maximal level of demand it can possibly serve. To determine a lower bound, note that it is not an equilibrium that total capacity is below total demand. In such a situation both firms would have an incentive to increase their capacities until total capacity equals total demand, as firms remain local monopolies up to this level. Hence, the total equilibrium capacity is at least 4, but not above 8.[8] In the next step we argue that total capacity is equal to total demand in equilibrium when there is no demand uncertainty. This result of no excess capacities when demand is certain resembles Kreps et al.

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[7]A prove of this case is available and can be provided upon request.

[8]For this argument we implicitly focus on pure strategy equilibria in the capacity game. These situations could arise as outcomes in a mixed strategy equilibrium with non-zero probability.

(1983). In particular, it is an equilibrium that each firm has two units of capacity.[9]

For this argument we have to derive the profit levels for deviating capacity choices. We have already established the equilibrium and expected profits for the symmetric capacity configurations (the reference cases of 2 and 4 units of capacity by each firm are in section 4, and the case of 3 units each is in section 5).

Let us now consider that total capacity is larger than 4 and thus exceeds demand. Consider that one firm, say L, has the weakly larger capacity, denoted by l. Denoting the capacity of firm R with r, the cases that are left to inspect are $\{l = 3, r = 2\}$ and $\{l = 4, r = 2\}$.

For the subsequent pricing game, assume that firms can only play uniform price vectors. We argue at the end of this section that the analysis naturally extends to all mixed strategy equilibria in weakly increasing prices.

Let us start with the capacity configuration $\{l = 3, r = 2\}$. When its competitor plays uniform prices, each firm can guarantee itself a minimum profit of serving its closest customer(s) at a price of v. For firm L this minimum profit is $v - c + v - 2c = 2v - 3c$, and for firm R it is $v - c$. These profits define the lowest prices each firm is willing to play as uniform prices. For firm L the lowest price is defined by the indifference in terms of profits between the highest and the lowest price: $2v - 3c = 3\underline{p}_L - 6c$. This yields $\underline{p}_L = \frac{2}{3}v + c$. Similarly, the lowest price of firm R is derived from $v - c = 2\underline{p}_R - 3c$, which yields $\underline{p}_R = \frac{v}{2} + c$. The prices derived for the two firms are not identical, and the lowest price that is played by both firm is defined by $\max(\underline{p}_L, \underline{p}_R) = \underline{p}_L \equiv \underline{p}$. Thus, at $\underline{p}$ firm R makes a profit $\pi_R(\underline{p})$, which exceeds the profit at a price of v if L does not play v with positive probability (the latter corresponds to a mass point at v). Hence, a mass point at v for firm L is required such that the equilibrium profits are given by $\pi_L(v) = 2v - 3c$ and $\pi_R(\underline{p}_L = \underline{p}) = \pi_R(v) = \frac{4}{3}v - c$. Comparing these profits with the profits at capacity levels of $\{l = 3, r = 3\}$ shows that each firm has a weak incentive to reduce the capacity from 3 to 2. The incentive is strict if capacity is costly. Analogously, with capacities $\{l = 2, r = 2\}$ a firm has no incentive to increase the capacity from 2 to 3 as the profit does not increase. See table 1 for a summary of the profits.

For completeness, let us check whether more drastic deviations from capacities $\{l = 2, r = 2\}$ are profitable by considering the case $\{l = 4, r = 2\}$. In this case firm R cannot unilaterally ensure for itself a positive profit. Instead, as before firm L can guarantee itself a profit of $2v - 3c$. Hence, also the lowest price L is willing to play is $\underline{p} = \frac{2}{3}v + c$ as before. This is the maximum of the lowest prices, as R would be willing to play lower prices because it has no "guaranteed" positive profit. This again yields expected profits of $\pi_L(v) = 2v - 3c$. As before, the profit of R is defined by the profit it obtains at $\underline{p}$, the lowest price L is willing to set. Hence, $\pi_R(\underline{p}) = \frac{4}{3}v - c$ must be the expected profit R obtains at any price it plays, also at v. Note that this again requires a mass point at v for L, as otherwise playing v would yield a zero profit for R and would thus not occur.

In summary, if one considers a situation of 2 capacity units for each firm, no firm has an incentive to lower its capacity. This would simply leave valuable demand unserved. Also, no firm has an incentive to increase capacity. A firm that has a lager capacity, while the other

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[9]Given the different transport costs, this is the Pareto-dominating outcome for firms and also maximizes total surplus.

firm has a capacity of 2 does still make the same profit – gross of capacity costs, as with 2 units of capacity.

		Firm R		
Capacities	1	2	3	4
1	$v - c,\, v - c$	$v - c,\, 2v - 3c$	$v - c,\, 3v - 6c$	$\frac{3}{4}v,\, 3v - 6c$
2	$2v - 3c,\, v - c$	$2v - 3c,\, 2v - 3c$	$\frac{4}{3}v - c,\, 2v - 3c$	$\frac{4}{3}v - c,\, 2v - 3c$
3	$3v - 6c,\, v - c$	$2v - 3c,\, \frac{4}{3}v - c$	$v - c,\, v - c$	$\frac{3}{4}(v - c),\, v - c$
4	$3v - 6c,\, \frac{3}{4}v$	$2v - 3c,\, \frac{4}{3}v - c$	$v - c,\, \frac{3}{4}(v - c)$	$4c,\, 4c$

Table 1: Profits for different capacity levels when total demand is 4

To understand how capacity levels of 3 units per firm can arise when total demand is 4, consider that there are two possible states of demand: In one state total demand is 4 units, in the other state total demand is 6 units because the 4 customers each demand 1.5 units. Assume that if the demand state is 6 units and each firm can only supply less than 3 units, but more than 1.5 units, it will partially serve the second customer up to its capacity limits. Both states occur with positive probability. As argued before, if the state of a demand of 4 units would occur with certainty, the capacity choice in equilibrium would be 2 units per firm. Analogously, if the state of a demand of 6 units would occur with certainty, the capacity choice in equilibrium would be 3 units per firm. If there is uncertainty, the capacity levels $\{l = 3, r = 3\}$ are an equilibrium as long as the high demand state is sufficiently likely. Note that having a capacity of 3 instead of 2 does not reduce the profits (gross of capacity costs) for the firm with the higher capacity. Nevertheless, a trade-off occurs as each firm can increase its profit in case of demand of 4 by reducing the capacity from 3 to 2, if the other firm has a capacity of 3 units. The profit increases by $\left(\frac{4}{3}v - c\right) - (v - c) = \frac{v}{3}$. This provides an incentive to choose a capacity of 2 instead of 3. In case of high demand, reducing the capacity from 3 to 2 when the other firm has a capacity of 3 reduces the sales to customers to which one could charge a monopoly price of v as there are no excess capacities. The profit loss from reducing the capacity by one is $v - 2c$. If the probability for the low demand state is α and that for the high demand state is $1 - \alpha$, there is an equilibrium in which both firms choose a capacity of 3 if $\alpha \frac{v}{3} < (1 - \alpha)(v - 2c) \Leftrightarrow \alpha < \frac{3v - 6c}{4v - 6c}$. For instance, for $v = 7c$, this yields $\alpha < \frac{15}{22}$.

In summary, when firms choose their capacities in view of uncertain demand, firms trade-off additional sales in high demand states with lower prices in lower demand states, due to overcapacities. The resulting capacity choices tend to yield excess capacity in the low demand states, and no excess capacity in the high demand states.

So far, we have assumed that firms only play uniform price vectors and computed the implied equilibrium profits. The analysis naturally extends to all mixed strategy equilibria in weakly increasing prices, as the equilibrium profits are computed in the same way. To see this, recall from section 5 that in these equilibria the lowest and highest prices in the price supports are played (also) as uniform prices. When charging the closest customer the highest price v, the requirement of weakly increasing prices implies a price of v for the more distant customers. The lowest price $\underline{p}$ is also played with joint density across all customers.

Otherwise, the price supports of the two firms would have non-overlapping areas, where one firm would charge prices that are below the lowest price of the other firm for some customer.[10] A caveat applies for this extension as we have not formally checked that playing weakly increasing prices is always an equilibrium in case of asymmetric capacity configurations. We, however, conjecture that the logic of the symmetric case carries over such that an equilibrium in weakly increasing prices can always be constructed.[11]

8   Conclusion

Strategic uncertainty and inefficient competition.   We have characterized a competitive equilibrium with prices that weakly increase in the costs of serving the different customers. This, together with limited overcapacities, yields the outcome that a firm typically serves its closest customers (its "home market"). However, intermediate customers are sometimes served by a more distant firm with higher costs, although there is a firm with lower costs and free capacities. Importantly, this occurs when location and customer specific pricing is feasible and firms do price discriminate in equilibrium. The reason is that price competition in the presence of capacity constraints results in unstable prices. As one competitor does not know which prices the other competitor will ultimately charge, there is strategic uncertainty and different prices are charged each time.

   The result of cost inefficiency due to strategic uncertainty arises, although we have tilted the model towards efficient supplies. In particular, we allow firms to price discriminate according to location, assume efficient rationing, and focus on equilibria with weakly increasing prices, such that the costs tend to be reflected in the price schedule. We conjecture that alternative rationing rules can yield even more inefficiencies. For instance, more chaotic price patterns may arise when prices and costs are less well aligned.

Cross-supplies.   Cross-supplies between suppliers can reduce the allocative inefficiencies of chaotic price competition. However, from a welfare perspective cross-supplies are a two-sided sword. On the one hand, they can clearly increase efficiency by reducing costs. On the other hand, they can dampen competition as a supplier that anticipates to become a cross-supplier has less incentives to aggressively compete for the customers in the first place. We find that cross-supplies do not harm customers through higher prices if the cross-supplier makes no profit on its subcontract (or a sufficiently low one). As a consequence, subcontracts where the cross-supplier will sell to a competitor at marginal costs tend to be pro-competitive, while arrangements which foresee that the cross-supplier earns a significant profit on a cross-supply tend to dampen competition. What matters here is the profit obtained from an additional cross-supply, and not necessarily the overall profitability. A subcontract arrangement can

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[10]This could only be an equilibrium outcome if that customer is never served by the firm with the higher price independent of its price, in which case it would be indifferent to change its price support.

[11]The basic intuition is that if transport costs are not that important relative to v, then uniform prices arise as a special case of weakly increasing prices.

therefore be pro-competitive if the cross-supplier is remunerated upon signing a framework agreement with a fixed fee and in turn conducts cross-supplies at marginal costs.

Efficiency increasing cross-supplies may not always take place because firms fear additional competition. In particular, we have shown that when an almost capacity-constrained firm asks the unconstrained firm for a cross-supply, the unconstrained firm may deny this supply. The reason is that the cross-supply would endow the demanding firm with additional capacity, which can intensify competition for other customers.

Bertrand-Edgeworth arguments in competition policy. Various competition policy cases feature homogeneous products with significant transport costs for which location or customer based price discrimination is common. Several decisions make explicit references to Bertrand-Edgeworth models, but without taking geographic differentiation and customer specific pricing into account. For instance, in relation to the merger M.6471 OUTOKUMPU / INOXUM in 2012 the European Commission (Commission) noted that "*one of the main criticisms of the Notifying Party of this [Bertrand-Edgeworth] model of aggressive competition is that it tends to predict more competitive prices pre-merger than the observed pre-merger price*". The Commission acknowledged that there may be frictions which have not been accounted for: "*Customers may have other preferences for a specific firm (e.g. geographic proximity, preferences for a specific firms products based on quality concerns or experiences; or on-going business relationships or contracts etc)*".[12] Our model allows for both customer-firm specific (transport) costs as well as prices and thus can aid the analysis in future cases.

Our model can also help to assess whether firms compete with each other or coordinate their sales activities. In the assessment of the merger M.7009 HOLCIM / CEMEX WEST in 2014 the Commission argued "*that the most likely focal point for coordination in the cement markets under investigation would be customer allocation whereby competitors refrain from approaching rivals' customers with low prices. Under such a coordination scenario, the sizable transport costs for cement would lead to a general allocation of customers based on proximity to a given plant. The Commission has thus investigated the hypothesis that cement competitors might face limited incentives to enter significantly into competitors' geographic strongholds...*".[13] The Commission concludes that "*given the low level of differentiation across firms and the existing overcapacities, it is difficult to explain the observed level of gross margins as being the result of competitive interaction between cement firms.*"[14] As a supporting argument, the Commission refers to a Bertrand-Edgeworth model with constant marginal costs and uniform pricing.[15]

Our model makes several predictions which can be related to the above reasoning: Even with overcapacities of 50%, we find that in a competitive equilibrium firms may always exclusively serve their home markets, and that at prices above the costs of the closest competitor.

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[12]See paragraphs (725) and (407) in the Commission's decision in M.6471 OUTOKUMPU / INOXUM, and more generally Annex IV of the decision for the Bertrand-Edgeworth modelling.

[13]See paragraphs (167) and (168).

[14]Paragraph (178).

[15]See the European Commission decision M.7009 HOLCIM / CEMEX WEST, fn. 195.

Moreover, firms set high prices in the home markets of rival firms, although a unilateral undercutting there seems rational in view of their overcapacities. Such a pattern is difficult to reconcile with previous models of competition. On the one hand, without capacity constraints, the logic of asymmetric Bertrand competition predicts prices equal to the marginal costs of the second most efficient firm. On the other hand, the typical Bertrand-Edgeworth model with capacity constraints, but uniform costs and uniform pricing, does not explain spatial price differentiation and customer allocation.

Moreover, both the Bertrand model with customer specific costs and prices, but without capacity constraints, as well as the homogeneous Bertrand-Edgeworth model tend to predict significantly lower competitive margins than our model, which can incorporate all of these market features simultaneously.[16] When using an over-simplified model, a competition authority may thus wrongly conclude that the observed market outcome cannot be the result of competition, but rather of coordination among the suppliers. To answer the question whether suppliers are indeed coordinating or competing, the new model – which allows for geographic differentiation, location specific pricing and capacity constraints at the same time – could therefore improve the reliability of competition policy assessments.

There is plenty of scope for further research. On the more theoretical frontier, avenues for future research include the introduction of a continuous demand structure as well as alternative rationing rules. On the more applied frontier, reformulating the model to allow for more than two firms and simulating the effects of mergers in such a setting appears to be of particular interest.

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[16]See Annex II for sample calculations of unexplained excess margins when using the different models.

Annex I: Explicit price strategy for strictly increasing prices

In this annex we present an example of an equilibrium price strategy for the case of strictly increasing prices ($5c < v < 7c$), as described in proposition 2. In particular, we illustrate that a firm can draw prices from a joint distribution such that marginal distributions are F_{12} for the two closest and F_{34} for the two most distant prices, as defined in (7) and (8), and the resulting price vectors are always weakly increasing with $p_1 = p_2 \le p_3 = p_4$.

Suppose that firm L initially draws a price $p_1 \in [\underline{p}, v]$ from the distribution function F_{12}. It then sets $p_2 = p_1$ as also p_2 must be played according to the marginal distribution F_{12} and $p_1 = p_2$ is a requirement of the equilibrium strategies.

Recall that $F_{12}(p)$ equals $F_{34}(p)$ for $p \in [4c, v]$, which is only consistent with uniform price vectors. Firm L thus also sets the other prices p_3 and p_4 equal to p_1 in this interval.

For $p_1 \in [\underline{p}, 4c)$, firm L faces the problem that only playing uniform prices is not consistent with the marginal distributions as F_d first-order stochastically dominates F_c in that range. In particular, there is a price $\tilde{p} \in (\underline{p}, 4c)$, such that

$$f_{12}(\tilde{p}) = f_{34}(\tilde{p}),$$
$$f_{12}(p) > f_{34}(p) \text{ for } [\underline{p}, \tilde{p}),$$
$$f_{12}(p) < f_{34}(p) \text{ for } (\tilde{p}, 4c].$$

Small prices of close customers are played more often than small prices of distant customers. Firm L now can do the following:

1. For each realized price $p_1 \in [\underline{p}, \tilde{p})$,

    (a) with probability $\alpha \equiv f_{34}(p_1)/f_{12}(p_1)$ set uniform prices $p_1 = p_2 = p_3 = p_4$.

    (b) with probability $(1 - \alpha)$ set prices p_3 and p_4 in the interval $[\tilde{p}, 4c]$ according to the density function $f_{relocate} = \frac{f_{34}(p) - f_{12}(p)}{F_{34}(4c) - F_{12}(4c) - (F_{34}(\tilde{p}) - F_{12}(\tilde{p}))}$.

2. For each realized price $p_1 \in [\tilde{p}, 4c]$,

    (a) with probability 1 set uniform prices $p_1 = p_2 = p_3 = p_4$.

In case of low prices below $\tilde{p}$, the firm draws a higher price in the interval $[\tilde{p}, 4c]$ based on $f_{relocate}$ with probability $(1 - \alpha)$ according to step 1 (b). This density function is constructed in a way that density for distant prices is allocated from any point in the lower interval $[\underline{p}, \tilde{p})$ to the upper interval $[\tilde{p}, 4c)$ in proportion to the density $f_{34}(p) - f_{12}(p)$. This is the "missing" density when only uniform prices are played in response to realizations of $p_1 \in [\tilde{p}, 4c]$ with probability α according to step 2 (a) of the above rule. As a consequence, the distant prices materialize according to the marginal distribution function F_d. Note that the firm only plays strictly increasing prices in step 1(b), which happens with probability $\int_{\underline{p}}^{\tilde{p}} f_1(p)(1 - \alpha)dp = F_1(\tilde{p}) - F_3(\tilde{p})$. In summary, the joint distribution is characterized as follows: In the upper

part of the interval starting at $4c$, only uniform price vectors are played. Uniform prices are also often played in the lower part of the interval (based on step $2(a)$). In order to ensure that the different marginal densities f_{12} and f_{34} materialize in the lower interval $[\underline{p}, 4c)$, strictly increasing prices are generated in step $2(b)$ of the above rule.

Annex II: Unexplained excess margins when using an over-simplified model

Consider a market with customer-firm specific transport costs and pricing as well as capacity constraints. Without the Betrand-Edgeworth model developed in this article, one would need to use a pre-existing model of competition to predict the competitive market outcome. Such a traditional model would either disregard capacity constraints or heterogeneous transport costs (as done in the modeling of the European Commission in recent merger cases, see section 8). However, as we demonstrate below, these simplified models may predict competitive prices and profits which are significantly below those which the more appropriate model would predict. Thus, by disregarding one of these factors, a competition authority may wrongly conclude that the observed market outcome cannot derive from competition, but rather from coordination.

Model	Predicted profit	Unexplained excess margin $(v = 7c)$
Model of this article (no subcontracting)	$v - c$	0% (this reflects the "true" market outcome with capacity constraints and transport costs)
Bertrand-Edgeworth with uniform prices and uniform costs, based on accounting costs	$v - 1.75c$	$(v - c)/(v - 1.75c) - 1 = 14\%$
Bertrand-Edgeworth with uniform prices and uniform costs based on simple average costs	$v - 2.5c$	$(v - c)/(v - 2.5c) - 1 = 71\%$
Asymmetric Bertrand competition with unlimited capacity	$4c$	$(v - c)/4c - 1 = 50\%$

Table 2: Profit predictions of various models and excess margins relative to model with heterogeneous costs and capacity constraints (first line).

Table 2 lists the equilibrium profits which result on the basis of various models. *Main model* refers to the model analyzed in section 5, which accounts for firm-customer specific transport costs as well as capacity constraints and allows for customer specific prices. *Unexplained excess margin* is the fraction of the predicted profit of the main model over the prediction of the respective simplified model, minus one. Consider that one observes market data which is as predicted by the main model (that is assumed to appropriately reflect the market). The higher the actual margins are compared to those of a more simple competitive

model, the more likely one would conclude that the market outcome is not the result of competition, but may rather result from coordination. In the example depicted in Table 2, the actual competitive margins may be between 14 to 71 above the margins predicted by the homogeneous Betrand-Edgeworth model, depending on the employed cost measures in the more simple model.

Let us briefly sketch how we calculated the results presented in the table. We computed the unexplained excess margin as the ratio of the profit of the main model (assumed to reflect the "observed true" market outcome) and the profit of the simplified model, minus one. This measures how much the "observed true" margin is above the margin that can be explained with a (traditional) competitive model. Note that we use our main model without subcontracting here. With subcontracting, the predicted profits of our model can be even higher and range from $v - c$ to v. As a consequence, when subcontracting takes place but is not accounted for, the unexplained excess margins can be even higher.

Let us now explain the origin of the expressions in the different lines. The margin $(v - c)$ is taken from the the main model (see proposition 1). The margin range in case of subcontracting is taken from propositions 3 and 4.

For the Bertrand-Edgeworth models with uniform prices and costs, note that the residual profit, and thus equilibrium profit, is $v - k$, where k denotes the uniform costs of serving any customer. Here we consider two variants of how these "uniform" costs are computed in such a case where the true costs differ across customers:

1. Assuming that the actual market outcome is as characterized in proposition 1 yields accounting costs (as in the firm's management accounts) per unit of $k = (c + 2c \cdot 50\% + 3c \cdot 50\%)/2 = 1.75c$ for $v \geq 7c$;

2. Using simple average costs across all customers yields $k = (1c + 2c + 3c + 4c)/4 = 2.5c$. The implicit assumption is here that each customer is served by the same firm with the same probability.

For asymmetric Bertrand competition without capacity constraints, the profit is taken from the reference case in section 4.

References

Acemoglu, D., Bimpikis, K., and Ozdaglar, A., "Price and capacity competition." *Games and Economic Behavior*, Vol. 66 (2009), pp. 1–26.

Armstrong, M. and Vickers, J., "Competitive Price Discrimination." *RAND Journal of Economics*, Vol. 32 (2001), pp. 579–605.

Baake, P., Oechssler, J., and Schenk, C., "Explaining cross-supplies." *Journal of Economics*, Vol. 70 (1999), pp. 37–60.

Bertrand, J., "Théorie des richesses: revue des théories mathématiques de la richesse sociale par Léon Walras et reserches sur les principes mathématiques de la richesses par Augustin Cournot." *Journal des Savantes*, (1883), pp. 499–508.

Boccard, N. and Wauthy, X., "On the Nature of Equilibria when Bertrand meets Edgeworth on Hotelling's Main Street." (2016), mimeo.

Canoy, M., "Product differentiation in a Bertrand–Edgeworth duopoly." *Journal of Economic Theory*, Vol. 70 (1996), pp. 158–179.

Dasgupta, P. and Maskin, E., "The existence of equilibrium in discontinuous economic games, I: Theory." *The Review of Economic Studies*, Vol. 53 (1986), pp. 1–26.

Davidson, C. and Deneckere, R. J., "Excess Capacity and Collusion." *International Economic Review*, Vol. 31 (1990), pp. 521–41.

Edgeworth, F. Y., "The pure theory of monopoly." *Papers relating to Political Economy*, Vol. 1 (1925), pp. 111–142.

Fershtman, C. and Gandal, N., "Disadvantageous semicollusion." *International Journal of Industrial Organization*, Vol. 12 (1994), pp. 141–154.

Huff, N., "Horizontal Subcontracting in Procurement Auctions." *All Dissertations*, (2012), 917.

Kamien, M. I., Li, L., and Samet, D., "Bertrand Competition with Subcontracting." *RAND Journal of Economics*, Vol. 20 (1989), pp. 553–567.

Kreps, D. M., Scheinkman, J. A., et al., "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes." *Bell Journal of Economics*, Vol. 14 (1983), pp. 326–337.

Marion, J., "Sourcing from the Enemy: Horizontal Subcontracting in Highway Procurement." *The Journal of Industrial Economics*, Vol. 63 (2015), pp. 100–128.

Normann, H.-T. and Tan, E. S., "Effects of different cartel policies: evidence from the German power-cable industry." *Industrial and Corporate Change*, Vol. 23 (2014), pp. 1037–1057.

Sinitsyn, M., "Equilibria in a Capacity-Constrained Differentiated Duopoly." (2007), mimeo.

Sinitsyn, M., "Technical Note - Price Promotions in Asymmetric Duopolies with Heterogeneous Consumers." *Management Science*, Vol. 54 (2008), pp. 2081–2087.

Sinitsyn, M., "Price dispersion in duopolies with heterogeneous consumers." *International Journal of Industrial Organization*, Vol. 27 (2009), pp. 197–205.

Somogyi, R., "Bertrand-Edgeworth competition with substantial horizontal product differentiation." (2016), mimeo.

Spiegel, Y., "Horizontal Subcontracting." *RAND Journal of Economics*, Vol. 24 (1993), pp. 570–590.

Thisse, J. and Vives, X., "On the Strategic Choice of Spatial Price Policy." *American Economic Review*, Vol. 78 (1988), pp. 122–37.

Varian, H. R., "A model of sales." *American Economic Review*, Vol. 70 (1980), pp. 651–659.

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